# Earthquakes and Logarithms

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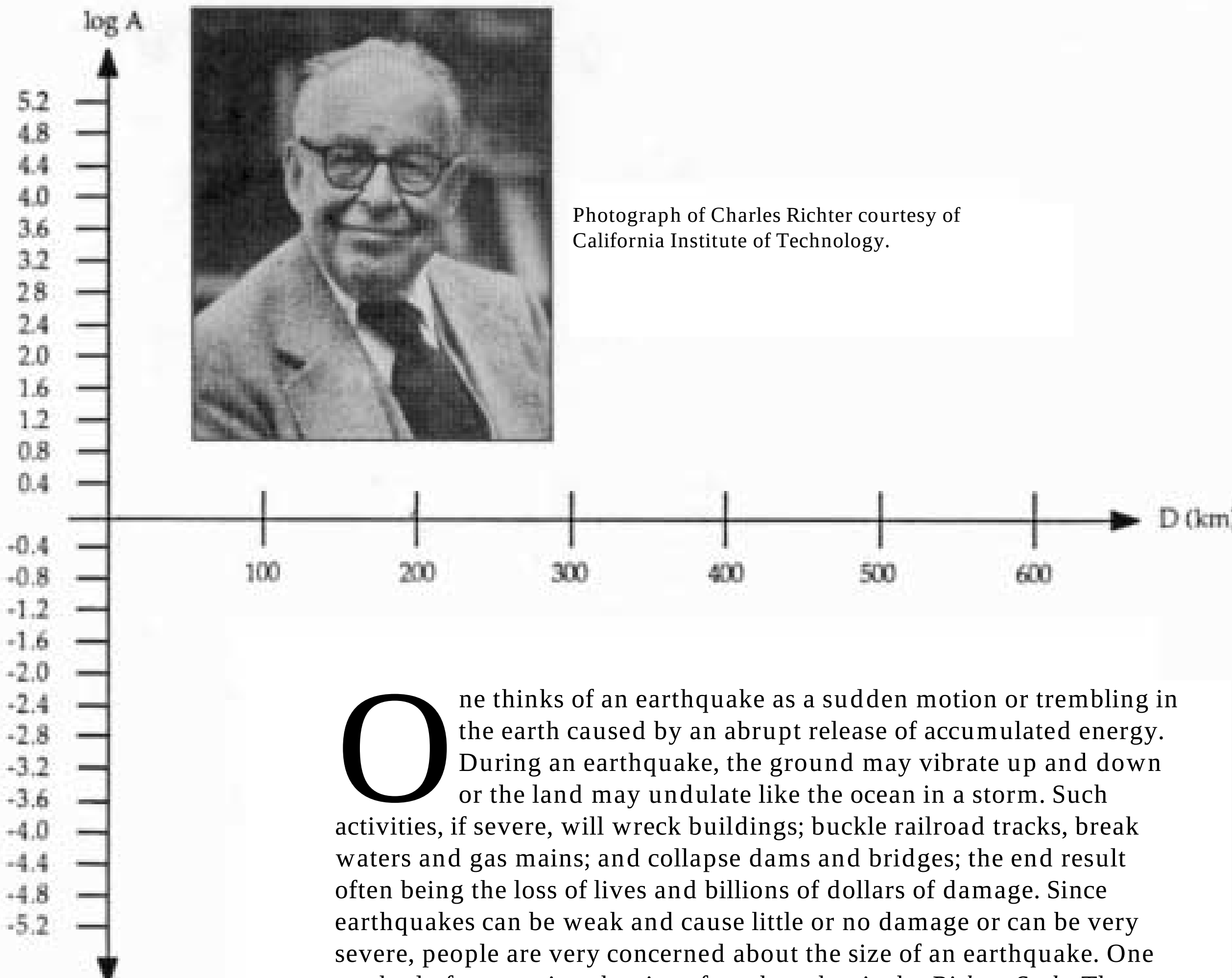


Photograph of Charles Richter courtesy of California Institute of Technology.

One thinks of an earthquake as a sudden motion or trembling in the earth caused by an abrupt release of accumulated energy. During an earthquake, the ground may vibrate up and down or the land may undulate like the ocean in a storm. Such activities, if severe, will wreck buildings; buckle railroad tracks, break waters and gas mains; and collapse dams and bridges; the end result often being the loss of lives and billions of dollars of damage. Since earthquakes can be weak and cause little or no damage or can be very severe, people are very concerned about the size of an earthquake. One method of comparing the size of earthquakes is the *Richter Scale*. The purpose of this Pull-Out Section is to show how mathematics is used in the Richter Scale and how mathematics played an important role in its development.

In the early '30s, Charles F. Richter, a *seismologist* (a person who studies earthquakes) at the California Institute of Technology, set out to develop a method of measuring the size of an earthquake. By means of mathematics and *seismograms* (a visual record produced by a *seismograph*, an instrument that records earthquakes), he developed in 1935 what is known as the Richter Scale for measuring the size of an earthquake. The discussion that follows has been simplified and many of the technical terms have been omitted.

**Figure** 1 below shows a portion of a seismogram for two earthquakes.

**Figure 1.**

The curve on the left indicates that there was very little movement of the earth, while on the right there was much movement. That portion of the curve on the left is said to have a small amplitude, while the one on the right has a large amplitude, where the *amplitude* is the vertical distance between a peak (or a valley) of the curve and a horizontal line formed if there were no earth movement. By observing many seismograms from southern California earthquakes, Richter verified what others previously had seen: the further an earthquake source is from a seismograph, the smaller will be the seismic wave amplitude. Stated another way, if two earthquakes have the same source and are recorded by the same seismograph, the stronger of the two earthquakes will produce larger amplitudes on the seismogram than will the weaker earthquake. If **Figure 1** represents the readings of two earthquakes having the same source, the earthquake recorded on the left side is weak since the amplitudes are nearly zero, while the one on the right has large amplitudes from a strong earthquake.

From these southern California earthquakes, Richter noted other features of earthquakes and he collected the following data:

1) the largest amplitude (measured in millimeters) of each earthquake, and

2) the distances (measured in kilometers) between the source of the earthquakes and the seismographs.

*Note:* To determine the source of an earthquake and the distance between the source of an earthquake and a seismograph involves much mathematics and many technical terms. Due to a lack of space, these items will not be discussed here.

The data that he secured is similar to the data presented in **Table 1** below. This data is not actual recorded data; it has been "chocolate coated" (or fudged) to illustrate how Richter used his data in his work.

**Table 1.**

| | Seismograph A | | | Seismograph B | | | Seismograph C | | | Seismograph D | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Earthquake | Distance | Largest Ampl. | | Dist. | Largest Ampl. | | Dist. | Largest Ampl. | | Dist. | Largest Ampl. | |
| | *D* | *A* | Log *A* | *D* | *A* | Log *A* | *D* | *A* | Log *A* | *D* | *A* | Log *A* |
| #1 | 20 | 794 | | 60 | 63 | | 180 | 16 | | 380 | 1.6 | |
| #2 | 180 | 2.5 | | 340 | 0.4 | | 380 | 0.25 | | 460 | 0.16 | |
| #3 | 340 | 0.16 | | 380 | 0.1 | | 460 | 0.06 | | 540 | 0.04 | |
| #4 | 20 | 6 | | 60 | 0.5 | | 180 | 0.13 | | 340 | 0.02 | |

## You Try It #1

On **Figure 2** below, for earthquakes #1, #2, and #3 in Table 1, plot the ordered pairs (*D*, *A*) where *D* equals the distance between the source of the earthquake and the seismograph and *A* equals the largest amplitude. Then connect the points with a "nice smooth curve" passing through the points for each of the given earthquakes.

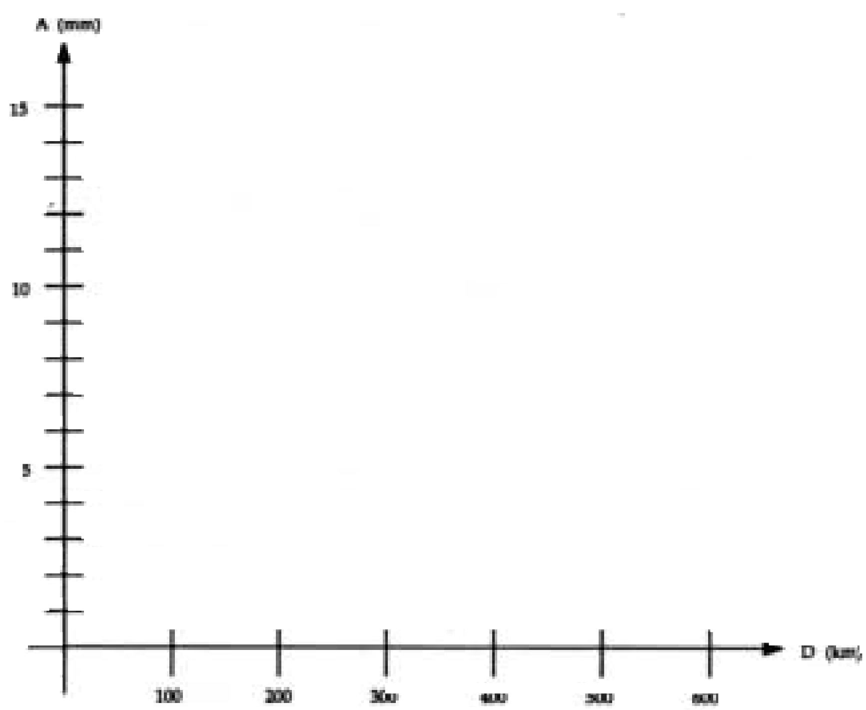


**Figure 2.**

Part of your graph for earthquake #1 should have extended far above the graph in Figure 2. Richter ran into similar problems as he noticed amplitudes of large earthquakes to be as much as 10 million times that of small ones. To compress this data, he computed log *A*, the base 10 logarithm of $A$, for each of the largest amplitudes $A$. Since $\log A < A$, the values of $\log A$ are much easier to fit on a graph.

## You Try It #2

**(a)** With a hand calculator find $\log A$ for each largest amplitude $A$ in Table 1.

**(b)** Round off each value of $\log A$ to the nearest tenth and enter that quantity into the space provided in Table 1.

**(c)** With this new data in Table 1, on **Figure 3** plot ($D$, $\log A$) and connect the points with a "nice smooth curve" for each of these earthquakes.

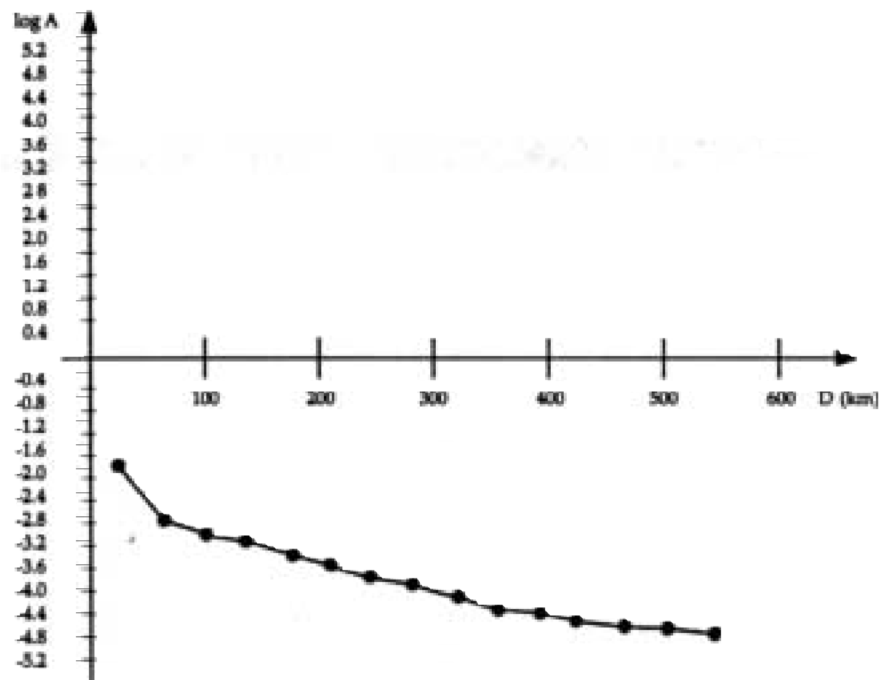


**Figure 3.**

The four curves in **Figure 3** should look as if they are parallel. It is this parallelism of curves from his collected data that Richter noted and used for developing a way of measuring the size of an earthquake. From **Table 1** or **Figure 3**, notice that for earthquakes #1 and #4, there are amplitude readings for both earthquakes at the common distance of 20km, 60km, and 180km. From this information we observe that:

At 20 km, $\log(794) - \log(6.0) \cong 2.9 - (0.8) = 2.1$,
At 60 km, $\log(63) - \log(0.5) \cong 1.8 - (-0.3) = 2.1$,
At 180 km, $\log(16) - \log(0.13) \cong 1.2 - (-0.9) = 2.1$.

## You Try It #3

**(a)** From **Table 1** or **Figure 3**, for each of the common distances of earthquakes #1 and #2, approximate $\log A_1 - \log A_2$, where $A_k$ is the largest amplitude of earthquake #$k$.

**(b)** As in **(a)**, make the same approximations for earthquakes #2 and #3.

**(c)** As in **(a)**, make the same approximations for earthquakes #2 and #4.

From the example above and from YOU TRY IT #3, we observe, as did Richter, that the difference of the logarithms of largest amplitudes of any two earthquakes is *independent* of the distance.

Richter then developed a "standard earthquake"; this is an earthquake that is used for comparison with other recorded earthquakes. (This is analogous to "par" for a golf course whereby a golfer compares his/her score as being above par, equal to par, or below par.) Richter used the data presented in **Table 2** below [Richter, p.342], as the "standard earthquake" and it is plotted in Figure 3 above.

**Table 2.**

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Dist. (km) | 20 | 60 | 100 | 140 | 180 | 220 | 260 | |
| log $A_0$ | –1.7 | –2.8 | –3.0 | –3.2 | –3.4 | –3.6 | –3.8 | |
| Dist. (km) | 300 | 340 | 380 | 420 | 460 | 500 | 540 | 580 |
| log $A_0$ | –4.0 | –4.2 | –4.4 | –4.5 | –4.6 | –4.7 | –4.8 | –4.9 |

This now give us Richter's definition of the magnitude of an earthquake:

$$M = \log A - \log A_0,$$

where $A$ is the largest amplitude for a given earthquake, $A_0$ is the amplitude of the standard earthquake, where both $A$ and $A_0$ are measure at the same distance. The value of $M$, often referred to as the "magnitude of an earthquake on the Richter Scale," is the number that characterizes the size of an earthquake and is independent of the location of the recording station.

Let's approximate the magnitude of earthquake #1 at the distances of 20km, 60km, 180km, and 380km using Figure 3 or Tables 1 and 2.

$$\begin{aligned} \text{20km: } M &= 2.9 - (-1.7) = 4.6, \\ \text{60km: } M &= 1.8 - (-2.8) = 4.6, \\ \text{180km: } M &= 1.2 - (-3.4) = 4.6, \\ \text{380km: } M &= 0.2 - (-4.4) = 4.6. \end{aligned}$$

Hence, earthquake #1 has a magnitude of $M = 4.6$ on the Richter Scale.

## You Try It #4

Approximate the magnitude of the three other earthquakes from Figure 3 or Tables 1 and 2.

## You Try It #5

Approximate the magnitude of an extremely weak earthquake whose largest amplitude is 0.0001 mm as recorded on a seismograph located 60 km from the source of the earthquake.

In YOU TRY IT #5, we see that it is possible to have an extremely weak earthquake that is measured with a negative number on the Richter Scale. In turn, an earthquake whose largest amplitude would be identical with that of the standard earthquake would measure $M = 0$ on the Richter Scale. The standard earthquake was intentionally chosen low enough so that the magnitudes of even weak recorded earthquakes will be positive on the Richter Scale. Even though the Richter Scale has no upper limit, no earthquakes larger than 8.9 have ever been recorded. Generally speaking, earthquakes have to attain a magnitude of at least 5.5 before severe damage occurs.

## You Try It #6

One of the purposes of the Richter Scale is to compare two (or more) earthquakes. What is wrong with the following argument: "That earthquake yesterday measured 4.2 on the Richter Scale. Its largest amplitude was twice that of last week's earthquake which measured 2.1 on the Richter Scale."

## You Try It #7

People have claimed that, "The 1906 San Francisco earthquake measured 8.3 on the Richter Scale." What is wrong with that argument? Or is it correct? If it is correct, how could that be so?

## You Try It #8

As indicated in YOU TRY IT #7, the 1906 San Francisco earthquake had a magnitude of 8.3. In 1985, Mexico City had an earthquake that registered 7.8 on the Richter Scale. Compare the strengths of these two earthquakes in terms of their largest amplitudes.

The next time you hear that an earthquake has occurred, keep in mind what its number on the Richter Scale means and how it was computed. Also, remember that mathematics plays an important role in the development of the formula for computing the Richter Scale.

I offer a special note of thanks to my son, Peter A. Lindstrom, Jr. As a junior chemistry major at Caltech in October 1987, he experienced the earthquakes that shocked southern California in October 1987. He often talked about his trips to see the Caltech seismographs, reshelving books in the library, classmates reactions to the earthquakes, etc. In early 1988, I told him about my plans to investigate and write an article about the Richter Scale. In March 1988 he found for me the Richter reference, the basis for this Pull-Out Section. His interest and help made this Pull-Out Section a reality. *Thank you, son.*

### References

Richter, C. F. 1958. *Elementary Seismology*. San Francisco: W.H. Freeman and Co.

## Answers to the "You Try Its"

**1**

See **Figure 2**.

**2b**

#1: 2.9, 1.8, 1.2, 0.2
#2: 0.4, –0.4, –0.6, –0.8
#3: –0.8, –1.0, –1.2, –1.4
#4: 0.8, –0.3, –0.9, –1.7

**3a**

At 180 km, 1.2 – (0.4) = 0.8
At 380 km, 0.2 – (–0.6) = 0.8

**b**

At 340 km, –0.4 – (–0.8) = 0.4
At 380 km, –0.6 – (–1.0) = 0.4
At 460 km, –0.8 – (–1.2) = 0.4

**c**

At 180 km, 0.4 – (–0.9) = 1.3
At 340 km, –0.4 – (–1.7) = 1.3

**4**

#2, 3.8: #3, 3.4; #4, 2.5.

**5**

At 60 km, $M = \log(0.001) - (-2.8) = -3 + 2.8 = -0.2$.

**6**

At 100 km, $4.2 = \log x - (-3.0)$, so that $\log x = 1.2$, or $x = 10_{1.2}$.
At 100km, $2.1 = \log y - (-3.0)$, so that $\log y = -0.9$, or $y = 10_{-0.9}$.
Then $x / y = 10_{1.2} / 10_{-0.9} = 10_{2.1} \cong 125.9$. Hence, $x \neq 2y$.

**7**

The Richter Scale was developed in 1935, 29 years AFTER the San Francisco earthquake! But from seismological records of that earthquake, seismologists have estimated that that earthquake would have recorded 8.3 on the Richter Scale.

**8**

At any distance,
$8.3 = \log x - \log A_0$ and $7.8 = \log y - \log A_0$, or,
$8.3 = \log(x / A_0)$ and $7.8 = \log(y / A_0)$, or,
$x / A_0 = 10_{8.3}$ and $y / A_0 = 10_{7.8}$.
Then $x / y = (x / A_0) / (y / A_0) = 10_{8.3} / 10_{7.8} = 10_{0.5}$
$= \sqrt{10} \cong 3.16$, or $x = 3.16y$.